Rewriting of Regular Path Queries: The first paper of the four Italians

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Based on the paper
"Rewriting of Regular Expressions and Regular Path Queries",
by D. Calvanese, G. De Giacomo, M. Lenzerini, M. Y. Vardi

VardiFest22: On the not so unusual effectiveness of Logic

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Italian last names of the form "_ardi"

- Bardi
- Cardi
- Dardi
- Gardi
- Lardi
- Nardi
- Pardi
- Sardi
- Tardi
- Zardi

"Vardi" is missing, but it really sounds like an Italian last name ...

Given:

- a query Q over a database with alphabet Σ
- k view definitions (where each Q_i is a query over Σ):

$$q_1 \doteq Q_1, \quad \dots \quad , q_k \doteq Q_k$$

can we re-express Q in terms of the views $q_1, \dots, q_k$?

Several applications, such as:

- Answer Q by relying only on the precomputed answers to the views?
- Process queries in LAV data integration system, where sources are characterized as views over the global schema Σ

Graph databases and regular path queries

A **graph database** is a labeled graph, where nodes represents objects and each edge represents the fact that a certain relation holds between two nodes.

A **regular path query** Q is specified through a regular language $L(Q)$ over the edge labels.

The **answer to** Q **over a database** DB is:
$\{(x, y) \mid \exists(x \stackrel{a_1}{\to} x_1 \cdots \stackrel{a_n}{\to} y)$ in DB s.t. $a_1 \cdots a_n$ is a word in $L(Q)\}$

Example: $(child^* \cdot friend) + (friend \cdot child^*)$

Rewriting of regular expressions

- $\Sigma_{\mathcal{E}} = \{e_1, \ldots, e_k\}$
- $re(e_i) = E_i$ r.e. over Σ

For a language ℓ over $\Sigma_{\mathcal{E}}$, we define

$$exp_{\Sigma}(\ell) = \bigcup_{e_1 \cdots e_n \in \ell} \{w_1 \cdots w_n \mid w_i \in L(re(e_i))\}$$

Example: $\Sigma_{\mathcal{E}} = \{e_1, e_2\}$
$re(e_1) = a + b, \quad re(e_2) = c{\cdot}d + f$
$\ell = (e_1{\cdot}e_2) + e_1$
$exp_{\Sigma}(\ell) = \{acd, af, bcd, bf, a, b\}$

Rewriting of regular expressions

Intuitively:

Given a r.e. E_0 over Σ, an alphabet $\Sigma_{\mathcal{E}} = \{e_1, \ldots, e_k\}$, and one r.e. $re(e_i)$ over Σ for each $e_i \in \Sigma_{\mathcal{E}}$, we aim at re-expressing E_0 by a combination of $re(e_1), \ldots, re(e_k)$.

Formally:

A formalism R for defining a language $L(R)$ over $\Sigma_{\mathcal{E}}$ is a **rewriting of** E_0 wrt $\mathcal{E}$ if

$$exp_{\Sigma}(L(R)) \subseteq L(E_0).$$

Computing the rewriting of a regular expression

Input: r.e. E_0 over Σ, $\Sigma_{\mathcal{E}} = \{e_1, \ldots, e_k\}$, set $\{re(e_1), \ldots, re(e_k)\}$ of r.e. over Σ

Output: automaton $R_{\mathcal{E},E_0}$ (maximal rewriting of E_0 wrt $\mathcal{E}$)

1. Construct a **deterministic** automaton $A_d = (\Sigma, S, s_0, \rho, F)$ accepting $L(E_0)$.
2. Let $A' = (\Sigma_{\mathcal{E}}, S, s_0, \rho', S - F)$, where

   $$s_j \in \rho'(s_i, e) \quad \text{iff} \quad \exists w \in L(re(e)) \text{ s.t. } s_j \in \rho^*(s_i, w).$$

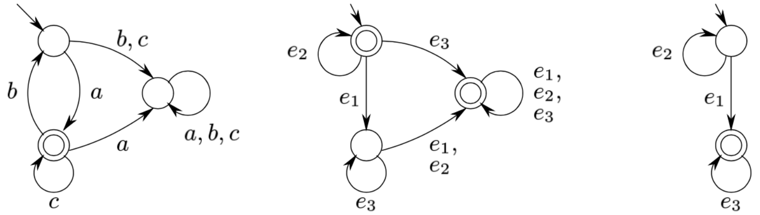
   A' **accepts** a $\Sigma_{\mathcal{E}}$-word $e_1 \cdots e_n$ iff there is a Σ-word in $exp_\Sigma(\{e_1 \cdots e_n\})$ **rejected** by A_d.
3. $R_{\mathcal{E},E_0}$ is $\overline{A'}$, i.e. the complement of A'.

Observation:

Any maximal rewriting of E_0 wrt $\mathcal{E}$ defines a **regular language**.

Rewriting of regular expressions (example)

$$E_0 = a\cdot(b\cdot a + c)^* \qquad \mathcal{E} = \{\underbrace{a}_{e_1}, \underbrace{a\cdot c^*\cdot b}_{e_2}, \underbrace{c}_{e_3}\}$$



A_d $\qquad$ A' $\qquad$ $\overline{A'} = R_{\mathcal{E},E_0}$

Complexity analysis

For a regular expression E_0 and a set $\mathcal{E}$ of regular expressions:

Generating the $\Sigma_{\mathcal{E}}$-maximal rewriting is in 2EXPTIME
(Two determinization steps.)

Checking existence of a nonempty rewriting is EXPSPACE-complete
(Hence the upper bound for generating the $\Sigma_{\mathcal{E}}$-maximal rewriting is optimal.)

Verifying the existence of an exact rewriting is 2EXPSPACE-complete

What came next

After this paper, Moshe visited regularly Rome, sometimes with Pam, often during the Christmas Holydays.

The group had such a great time, enjoying the city and working on several aspects of graph databases, including

- Answering queries using views (see Diego's talk)
- Two-way RPQs (containment, rewriting and answering)
- Conjunctive RPQs (containment, rewriting and answering)
- Two-way Conjunctive RPQs (containment, rewriting and answering)
- View-based containment
- ...

Moshe, I am looking forward to another ride in the traffic of Rome!